

Section 8.1 Solutions

Section 8.1: Systems of Linear Equations,
Substitution and Elimination

#1-6: Verify that the given values are solutions
to the system of equations.

$$3x + 2y = 11$$

1) $2x - y = 5$ (3,1)

TOP EQUATION

$$3(3) + 2(1) = 11$$

$$9 + 2 = 11$$

$$11 = 11$$

✓

BOTTOM EQUATION

$$2(3) - 1 = 5$$

$$6 - 1 = 5$$

$$5 = 5$$

✓

3) $\frac{2}{3}x + \frac{1}{2}y = 3$
 $2x - y = 4$ (3, 2)

TOP EQUATION

$$\frac{2}{3}(3) + \frac{1}{2}(2) = 3$$

$$\frac{6}{3} + \frac{2}{2} = 3$$

$$2 + 1 = 3$$

$$3 = 3$$

✓

BOTTOM EQUATION

$$2(3) - 2 = 4$$

$$6 - 2 = 4$$

$$4 = 4$$

✓

$$\begin{array}{l} x - y + z = 2 \\ 5) \quad 2x - 3y + 2z = 2 \\ \quad \quad x + y - 2z = 3 \end{array} \quad (3, 2, 1)$$

Top Equation

$$\begin{array}{l} 3 - 2 + 1 = 2 \\ 2 = 2 \quad \checkmark \end{array}$$

Middle Equation

$$\begin{array}{l} 2(3) - 3(2) + 2(1) = 2 \\ 6 - 6 + 2 = 2 \\ 2 = 2 \quad \checkmark \end{array}$$

Bottom Equation

$$\begin{array}{l} 3 + 2 - 2(1) = 3 \\ 3 + 2 - 2 = 3 \\ 3 = 3 \quad \checkmark \end{array}$$

#7 – 20: Solve each system of equations using the substitution method. If the system has no solutions say, that it is inconsistent. If the system has infinitely many solutions write your answer in the form

$$\{(x, y) | y = mx + b, x \text{ is any real number}\}$$

$$7) \quad y = 2x - 4$$

$$-2x + 3y = 0$$

First replace y in the bottom equation with $2x - 4$ & solve for x .

$$-2x + 3(2x - 4) = 0$$

$$-2x + 6x - 12 = 0$$

$$-4x - 12 = 0$$

$$\begin{array}{r} +12 \quad +12 \\ \hline \end{array}$$

$$\begin{array}{r} -4x \quad = \quad 12 \\ \hline \end{array}$$

$$x = 3$$

Second solve for y

$$y = 2x - 4$$

$$y = 2(3) - 4$$

$$y = 6 - 4$$

$$y = 2$$

ANSWER (3, 2)

$$3x + 2y = 11$$

$$9) \quad 2x + y = 5$$

First SOLVE Bottom Equation For y

$$\begin{array}{r} 2x + y = 5 \\ -2x \quad -2x \\ \hline y = -2x + 5 \end{array}$$

Second SUBSTITUTE INTO top Equation
& SOLVE For x

$$3x + 2(-2x + 5) = 11$$

$$3x - 4x + 10 = 11$$

$$-1x + 10 = 11$$

$$\begin{array}{r} -10 \quad -10 \\ \hline \end{array}$$

$$\frac{-1x}{-1} = \frac{1}{-1}$$

$$x = -1$$

Third SOLVE For y

$$y = -2x + 5$$

$$y = -2(-1) + 5$$

$$y = 2 + 5$$

$$y = 7$$

Answer (-1, 7)

~~the~~

$$11) \quad \begin{aligned} \frac{2}{3}x + \frac{1}{2}y &= 3 \\ 2x - y &= 4 \end{aligned}$$

First Rewrite Top Equation

$$6 \cdot \frac{2}{3}x + 6 \cdot \frac{1}{2}y = 6 \cdot 3$$

~~6x + 3y = 18~~

$$\frac{12}{3}x + \frac{6}{2}y = 18$$

$$4x + 3y = 18$$

Second Solve Bottom Equation For y

$$\begin{array}{r} 2x - y = 4 \\ \quad +y \quad +y \\ \hline 2x = y + 4 \\ -4 \quad \quad -4 \\ \hline \end{array}$$

$$2x - 4 = y$$

3rd : SUBSTITUTE INTO THE EQUATION CREATED IN STEP 1, SOLVE FOR X

$$4x + 3(2x - 4) = 18$$

$$4x + 6x - 12 = 18$$

$$10x - 12 = 18$$

$$\begin{array}{r} +12 \quad +12 \\ \hline 10x = 30 \\ \frac{10}{10} = \frac{30}{10} \end{array}$$

$$x = 3$$

11) CONCLUDED

4th SOLVE FOR y

$$y = 2x - 4$$

$$y = 2(3) - 4$$

$$y = 6 - 4$$

$$y = 2$$

ANSWER (3,2)

$$13) \begin{cases} \frac{4}{3}x + \frac{1}{5}y = 5 \\ x - y = -2 \end{cases}$$

1ST Rewrite top Equation

$$15 \cdot \frac{4}{3}x + 15 \cdot \frac{1}{5}y = 15 \cdot 5$$

$$\frac{60}{3}x + \frac{15}{5}y = 75$$

$$20x + 3y = 75$$

2ND SOLVE Bottom Equation For X

$$\begin{array}{r} x - y = -2 \\ + y \quad + y \\ \hline x = y - 2 \end{array}$$

3rd SUBSTITUTE & Solve For y

$$20(y-2) + 3y = 75$$

$$20y - 40 + 3y = 75$$

$$23y - 40 = 75$$

$$\begin{array}{r} 23y - 40 = 75 \\ +40 \quad +40 \\ \hline 23y = 115 \\ \frac{23y}{23} = \frac{115}{23} \end{array}$$

$$y = 5$$

13 CONCLUDED

4th Solve For X

$$X = y - 2$$

$$X = 5 - 2$$

$$X = 3$$

ANSWER (3, 2)

$$x - 2y = 7$$

$$15) \quad 2x - 4y = -3$$

1ST: SOLVE TOP EQUATION FOR y

$$\begin{array}{r} x - 2y = 7 \\ + 2y + 2y \\ \hline x = 2y + 7 \end{array}$$

2ND: SUBSTITUTE, AND ATTEMPT TO SOLVE FOR y

$$2(2y - 7) - 4y = -3$$

$$4y - 14 - 4y = -3$$

$$-14 = -3$$

All letters Dropped out. I Am Finished with my ALGEBRA.

Since A FALSE STATEMENT Remains

ANSWER: NO SOLUTION
OR INCONSISTENT

$$4x + 2y = 6$$

$$17) 12x + 6y = 15$$

1ST: SOLVE TOP EQUATION FOR y

$$\begin{array}{r} 4x + 2y = 6 \\ -4x \qquad -4x \\ \hline 2y = -\frac{4x}{2} + \frac{6}{2} \\ y = -2x + 3 \end{array}$$

2ND: SUBSTITUTE INTO BOTTOM EQUATION

$$12x + 6(-2x + 3) = 15$$

$$12x + (-12x) + 18 = 15$$

$$18 = 15$$

LETTERS DROP OUT, SO MY ALGEBRA IS FINISHED. STATEMENT IS FALSE

So ANSWER
NO SOLUTION
OR
INCONSISTENT

$$4x + 2y = 6$$

$$19) 12x + 6y = 18$$

1ST: Solve Top Equation For y

$$\begin{array}{r} 4x + 2y = 6 \\ -4x \qquad -4x \\ \hline \end{array}$$

$$\frac{2y}{2} = \frac{-4x}{2} + \frac{6}{2}$$

$$y = -2x + 3$$

2ND: SUBSTITUTE INTO BOTTOM EQUATION

$$12x + 6(-2x + 3) = 18$$

$$12x + (-12x) + 18 = 18$$

$$18 = 18$$

All letters cancel so I am
finished doing my Algebra.

19 CONCLUDED

ANSWER

you may write either

INFINITELY many solutions

OR

$\{ (x, y) \mid y = -2x + 3 \text{ For any Real number } x \}$

#21 – 36: Solve each system of equations using the elimination method. If the system has no solutions say, that it is inconsistent. If the system has infinitely many solutions write your answer in the form

$$\{(x, y) | y = mx + b, x \text{ is any real number}\}$$

(To do this solve either equation for y.

$$3x + 2y = 11$$

$$21) \begin{cases} 2x - y = 5 \end{cases} \times 2$$

1st) Multiply Bottom Equation By 2

$$\begin{aligned} 3x + 2y &= 11 \\ 4x - 2y &= 10 \end{aligned}$$

2nd) Add Together, Drop y, Solve For x

$$\begin{aligned} 3x + 2y &= 11 \\ 4x - 2y &= 10 \\ \hline 7x &= 21 \end{aligned}$$

$$x = 3$$

3rd) Solve For y

$$\begin{aligned} 3x + 2y &= 11 \\ 3(3) + 2y &= 11 \\ 9 + 2y &= 11 \end{aligned}$$

$$\begin{aligned} 9 + 2y &= 11 \\ -9 & \quad -9 \\ \hline 2y &= 2 \\ y &= 1 \end{aligned}$$

$$\boxed{\text{ANSWER}} \\ \boxed{(3, 1)}$$

$$4x - 2y = 7$$

$$23) (2x - 5y = -3) \rightarrow -4x + 10y = 6$$

1ST MULTIPLY BOTTOM EQUATION BY (-2)

2ND ADD TOGETHER, ELIMINATE X,
SOLVE FOR Y

$$4x - 2y = 7$$

$$-4x + 10y = 6$$

$$\hline 8y = 13 \frac{1}{8}$$

$$y = 13/8$$

3RD SOLVE FOR X, HEAVY CALCULATOR

$$4x - 2y = 7$$

$$4x - 2(13/8) = 7$$

$$4x - 13/4 = 7 \quad \left. \begin{array}{l} \\ +13/4 +13/4 \end{array} \right\} \text{CALCULATOR}$$

$$\hline 4x = \cancel{7} + 41/4$$

23 CONCLUDES

$$\frac{4x}{4} = \frac{41/4}{4}$$

$$x = 41/16 \text{ (CALCULATOR)}$$

ANSWER $(41/16, 13/8)$

$$\begin{array}{l} 5(5x + 2y = 3) \\ 25) (3x - 5y = 15) \cdot 2 \end{array}$$

1ST multiply TOP By 5
Bottom By 2

$$\begin{array}{r} 25x + 10y = 15 \\ 6x - 10y = 30 \end{array}$$

2ND combine drop out y , solve for x

$$\begin{array}{r} 25x + 10y = 15 \\ 6x - 10y = 30 \\ \hline 31x = 45 \\ \underline{31} \end{array}$$

$$x = 45/31$$

25 CONCLUDED

Solve for y

$$5x + 2y = 3$$

$$5\left(\frac{45}{31}\right) + 2y = 3$$

$$\begin{array}{r} \frac{225}{31} + 2y = 3 \\ -\frac{225}{31} \qquad \qquad -\frac{225}{31} \\ \hline \end{array}$$

$$\frac{2y}{2} = \frac{-132}{2}$$

$$y = -66/31$$

ANSWER $\left(\frac{45}{31}, -\frac{66}{31}\right)$

$$27) \begin{cases} \frac{2}{4}x + \frac{1}{2}y = 2 \\ x - y = 4 \end{cases}$$

1ST clear Fractions, Rewrite
TOP EQUATION

$$4 \cdot \frac{2}{4}x + 4 \cdot \frac{1}{2}y = 4 \cdot 2$$

$$\frac{8}{4}x + \frac{4}{2}y = 8$$

$$\frac{2x}{2} + \frac{2y}{2} = \frac{8}{2}$$

$$x + y = 4$$

2ND PAIR with Bottom, combine
to drop y, solve for x

$$x + y = 4$$

$$x - y = 4$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

3rd Solve For y:

$$x + y = 4$$

$$4 + y = 4$$

$$\begin{array}{r} -4 \quad -4 \\ \hline \end{array}$$

$$y = 0$$

ANSWER (4, 0)

$$29) \begin{cases} \frac{4}{3}x + \frac{1}{5}y = 5 \\ x - y = -2 \end{cases}$$

1ST REWRITE TOP EQUATION
WITHOUT FRACTIONS

$$15 \cdot \frac{4}{3}x + 15 \cdot \frac{1}{5}y = 15 \cdot 5$$

$$\frac{60}{3}x + \frac{15}{5}y = 75$$

$$20x + 3y = 75$$

2ND STACK EQUATIONS

$$20x + 3y = 75$$

$$3(x - y = -2)$$

3RD MULTIPLY BOTTOM BY 3,
DROP Y, SOLVE X.

29 CONCLUDED

$$\begin{array}{r} 20x + 3y = 75 \\ 3x - 3y = -6 \end{array}$$

$$\begin{array}{r} 23x \qquad = 69 \\ \hline 23 \end{array}$$

$$x = 3$$

4th solve for y

$$20x + 3y = 75$$

$$20(3) + 3y = 75$$

$$\begin{array}{r} 60 + 3y = 75 \\ -60 \qquad -60 \end{array}$$

$$3y = 15$$

$$y = 5$$

ANSWER ~~(3, 5)~~ (3, 5)

$$-5(x - 2y = 7)$$

$$31) 5x - 10y = -3$$

1st multiply top by -5, Add together

$$-5x + 10y = -35$$

$$5x - 10y = -3$$

$$0 = -38$$

Letters cancel, so I am finished with my algebra.

STATEMENT IS FALSE

So

ANSWER

No Solution

OR

INCONSISTENT

$$-2(4x - 2y = 6)$$

$$33) \quad 8x - 4y = 12$$

1st multiply top by (-2), combine

$$-8x + 4y = -12$$

$$8x - 4y = 12$$

$$0 = 0$$

All letters drop out so
I am finished with my
Algebra.

Since a TRUE STATEMENT
Remains it is sufficient
to write

ANSWER!

Infinitely many
Solutions

33) The other solution is optional and is not required. To generate the optional solution pick any equation & solve for y. They all will give the same answer.

$$\frac{4x - 2y}{-2} = \frac{6}{-2}$$

$$\begin{array}{r} -2x + y = -3 \\ +2x \qquad +2x \\ \hline y = 2x - 3 \end{array}$$

Now write solution

ANSWER $\{(x, y) \mid y = 2x - 3\}$
FOR ANY REAL $\neq x$

$$4x + 2y = 6$$

$$35) (2x + y = 3) \cdot (-2)$$

1st multiply Bottom Equation By (-2)

$$\begin{array}{r} 4x + 2y = 6 \\ -4x - 2y = -6 \end{array}$$

2nd COMBINE EQUATIONS


$$\begin{array}{r} 4x + 2y = 6 \\ -4x - 2y = -6 \\ \hline 0 = 0 \end{array}$$

3rd: All letters cancel ~~out~~ AND a true statement remains

TWO CORRECT ANSWERS, both will be acceptable on a test

Answer 1: Infinitely many solutions

Answer 2: $\{(x, y) \mid y = -2x + 3 \text{ for any real number } x\}$

Solve  Bottom Eqn. For y to get this.

37)

$$-x + 2y - z = -17$$

$$2x - y + z = 21$$

$$3x + 2y + z = 19$$

(pair the middle equation with the other 2 and drop out the y's)

STEP 1

$$\begin{array}{l} 2(2x - y + z = 21) \\ -x + 2y - z = -17 \end{array}$$



$$\begin{array}{r} 4x - 2y + 2z = 42 \\ -x + 2y - z = -17 \\ \hline 3x + z = 25 \end{array}$$

$$\begin{array}{l} 2(2x - y + z = 21) \\ 3x + 2y + z = 19 \end{array}$$



$$\begin{array}{r} 4x - 2y + 2z = 42 \\ 3x + 2y + z = 19 \\ \hline 7x + 3z = 61 \end{array}$$

#37)
STEP 2

$$-3(3x + z = 25)$$

$$7x + 3z = 61$$



$$-9x - 3z = -75$$

$$7x + 3z = 61$$

$$-2x \quad /-2 = -14 \quad /-2$$

$$x = 7$$

Solve for z:

$$3x + z = 25$$

$$3(7) + z = 25$$

$$21 + z = 25$$

$$z = 4$$

Solve for y: $3x + 2y + z = 19$

$$3(7) + 2y + 4 = 19$$

$$21 + 2y + 4 = 19$$

$$2y + 25 = 19$$

$$2y = -6$$

$$y = -3$$

Answer

$(7, 4, -3)$

39)

$$2x + y - 5z = -11$$

$$-x + y + 2z = 7$$

$$x - 3y + z = -2$$

(pair the middle equation with the other 2 and drop out the x's)

STEP 1

$$2(-x + y + 2z = 7)$$
$$2x + y - 5z = -11$$



$$\begin{array}{r} -2x + 2y + 4z = 14 \\ 2x + y - 5z = -11 \\ \hline 3y - z = 3 \end{array}$$

$$\begin{array}{r} -x + y + 2z = 7 \\ x - 3y + z = -2 \\ \hline -2y + 3z = 5 \end{array}$$

STEP 2

$$3(3y - z = 3)$$
$$-2y + 3z = 5$$



$$\begin{array}{r} 9y - 3z = 9 \\ -2y + 3z = 5 \\ \hline 7y = 14 \\ y = 2 \end{array}$$

39) CONCLUDED

Solve For z

$$-2y + 3z = 5$$

$$-2(2) + 3z = 5$$

$$-4 + 3z = 5$$

$$3z = 9$$

$$z = 3$$

Solve For X

$$X - 3y + z = -2$$

$$X - 3(2) + 3 = -2$$

$$X - 6 + 3 = -2$$

$$X - 3 = -2$$

$$+3 \quad +3$$

$$X = 1$$

Answer (1, 2, 3)

41)

$$2x + 2y - z = 2$$

$$x - 3y + z = -28 \quad \text{start anyway your like}$$

$$-x + y + 5z = 24$$

PAIR Bottom with other Two Drop X

STEP 1:

$$2(-x + y + 5z = 24)$$

$$2x + 2y - z = 2$$



$$-2x + 2y + 10z = 48$$

$$2x + 2y - z = 2$$

$$4y + 9z = 50$$

$$-x + y + 5z = 24$$

$$x - 3y + z = -28$$

$$\frac{-2y + 6z}{2} = \frac{-4}{2}$$

Divide By 2 to make # smaller

$$-1y + 3z = -2$$

STEP 2

$$4y + 9z = 50$$

$$4(-1y + 3z = -2)$$



$$4y + 9z = 50$$
$$-4y + 12z = -8$$

$$21z = 42$$

$$z = 2$$

41 concluded

Solve For y

$$4y + 9z = 50$$

$$4y + 9(2) = 50$$

$$4y + 18 = 50$$

$$4y \div 4 = 32 \div 4$$

$$y = 8$$

Solve For x

$$x - 3y + z = -28$$

$$x - 3(8) + 2 = -28$$

$$x - 24 + 2 = -28$$

$$x - 22 = -28$$

$$\begin{array}{r} x - 22 = -28 \\ +22 \quad \quad +22 \\ \hline x = -6 \end{array}$$

ANSWER (-6, 8, 2)

43)

$$4x + 4y + 4z = 12$$

$$4x - 2y - 8z = -12$$

start anyway you like

$$5x + 3y + 8z = 21$$

STEP 1

PAIR MIDDLE with other two, Drop z

$$\begin{array}{l} 4x - 2y - 8z = -12 \\ (4x + 4y + 4z = 12) \cdot 2 \end{array}$$



$$\begin{array}{l} 4x - 2y - 8z = -12 \\ 8x + 8y + 8z = 24 \\ \hline 12x + 6y = 12 \end{array}$$

$$12x + 6y = 12$$

Divide by 6

$$2x + y = 2$$

$$\begin{array}{l} 4x - 2y - 8z = -12 \\ 5x + 3y + 8z = 21 \\ \hline 9x + y = 9 \end{array}$$

STEP 2:

$$\begin{array}{l} -1(2x + y = 2) \\ 9x + y = 9 \end{array}$$



$$\begin{array}{r} -2x - y = -2 \\ 9x + y = 9 \\ \hline 7x = 7 \end{array}$$

$$x = 1$$

Solve For y

$$\begin{array}{l} 2x + y = 2 \\ 2(1) + y = 2 \\ 2 + y = 2 \\ y = 0 \end{array}$$

Solve For z

$$\begin{array}{l} 4x + 4y + 4z = 12 \\ 4(1) + 4(0) + 4z = 12 \\ 4 + 4z = 12 \\ 4z = 8 \\ z = 2 \end{array}$$

ANSWER

(1, 0, 2)

45)

$$2x - 2y + 3z = 6$$

$$4x - 3y + 2z = 0$$

$$-2x + 3y - 7z = 1$$

(pair the bottom equation with the other 2
and drop out the x's)

STEP 1:

$$\begin{array}{r} -2x + 3y - 7z = 1 \\ 2x - 2y + 3z = 6 \\ \hline \end{array}$$

$$y - 4z = 7$$

$$\begin{array}{r} 2(-2x + 3y - 7z = 1) \\ 4x - 3y + 2z = 0 \end{array}$$



$$\begin{array}{r} -4x + 6y - 14z = 2 \\ 4x - 3y + 2z = 0 \\ \hline \end{array}$$

$$3y - 12z = 2$$

STEP 2:

$$\begin{array}{r} -3(y - 4z = 7) \\ 3y - 12z = 2 \end{array}$$

$$\begin{array}{r} -3y + 12z = -21 \\ 3y - 12z = 2 \\ \hline \end{array}$$

$$0 = -19$$

ANSWER
INCONSISTENT
OR
NO SOLUTION

ALL LETTERS CANCEL, SO I AM DONE WITH ALGEBRA.
FALSE STATEMENT REMAINS

47)

$$2x + y - z = -2$$

$$x + 2y - z = -9$$

$$x - 4y + z = 1$$

(pair the bottom equation with the other 2 and drop out the z's)

STEP 1

$$\begin{array}{r} x - 4y + z = 1 \\ 2x + y - z = -2 \\ \hline 3x - 3y = -1 \end{array}$$

$$\begin{array}{r} x - 4y + z = 1 \\ x + 2y - z = -9 \\ \hline 2x - 2y = -8 \\ \text{DIVIDE BY 2} \end{array}$$

$$x - y = -4$$

STEP 2:

$$\begin{array}{r} 3x - 3y = -1 \\ -3(x - y = -4) \end{array}$$

↓

$$\begin{array}{r} 3x - 3y = -1 \\ -3x + 3y = 12 \\ \hline 0 = 11 \end{array}$$

ANSWER:
INCONSISTENT
OR
NO SOLUTION

ALL LETTERS DROP OUT SO ALGEBRA IS FINISHED.

49)

$$x - y - z = 1$$

$$-x + 2y - 3z = -4$$

$$3x - 2y - 7z = 0$$

(pair the middle equation with the other 2 and drop out the y's)

STEP 1:

$$-x + 2y - 3z = -4$$

$$2(x - y - z = 1)$$



$$-x + 2y - 3z = -4$$

$$2x - 2y - 2z = 2$$

$$x - 5z = -2$$

$$-x + 2y - 3z = -4$$

$$3x - 2y - 7z = 0$$

$$2x - 10z = -4$$

Divide By 2

$$x - 5z = -2$$

STEP 2

$$-1(x - 5z = -2)$$

$$x - 5z = -2$$

$$-x + 5z = 2$$

$$+x - 5z = -2$$

$$0 = 0$$

ANSWER
INFINITELY
MANY
SOLUTIONS

ALL VARIABLES CANCEL, SO ALGEBRA
IS COMPLETE.

51)

$$x + 2y - z = 8$$

$$-x - 3y + z = 23$$

$$x + y + 5z = 39$$

(pair the middle equation with the other 2 and drop out the z's)

STEP 1

$$-x - 3y + z = 23$$

$$x + 2y - z = 8$$

$$\begin{array}{r} 4y = 31 \\ -1 \quad -1 \end{array}$$

$$y = -31$$

$$-5(-x - 3y + z = 23)$$

$$x + y + 5z = 39$$

↓

$$5x + 15y - 5z = -115$$

$$x + y + 5z = 39$$

$$6x + 16y = -76$$

DIVIDE BY 2

$$3x + 8y = -38$$

STEP 2

$$y = -31$$

$$3x + 8y = -38$$

Solve BY SUBSTITUTION

51 concluded

Solve for X

$$3x + 8(-31) = -38$$

$$\begin{array}{r} 3x - 248 = -38 \\ +248 \quad +248 \end{array}$$

$$3x/3 = 210/3$$

$$x = 70$$

Solve for z

$$-x - 3y + z = 23$$

$$-70 - 3(-31) + z = 23$$

$$-70 + 93 + z = 23$$

$$z + 23 = 23$$

$$\begin{array}{r} z + 23 = 23 \\ -23 \quad -23 \end{array}$$

$$z = 0$$

Answer (70, -31, 0)